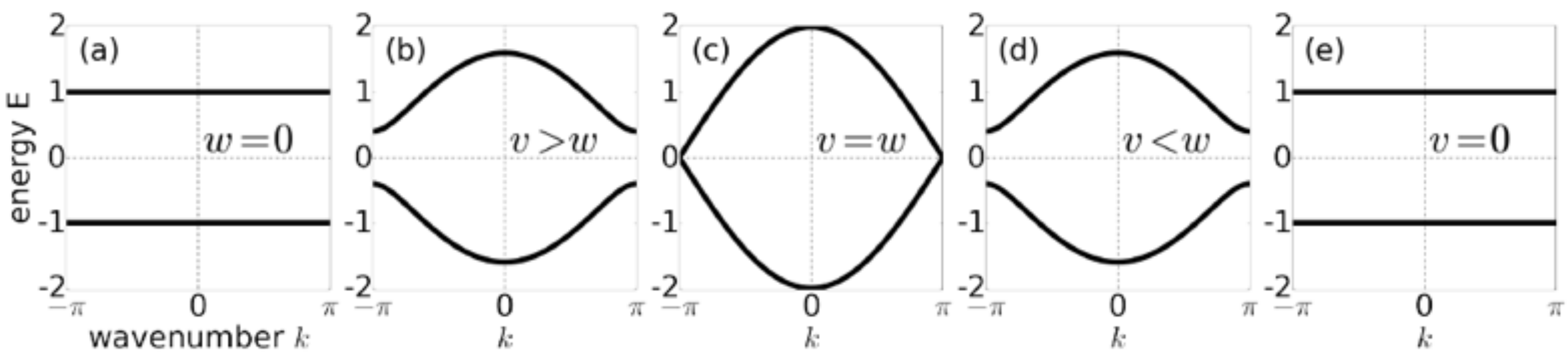
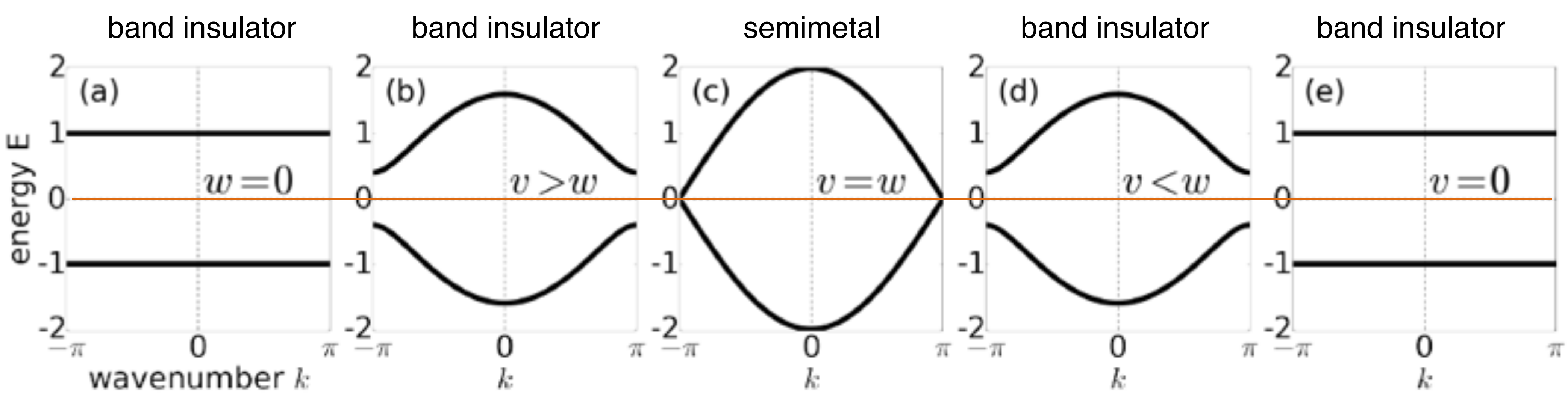
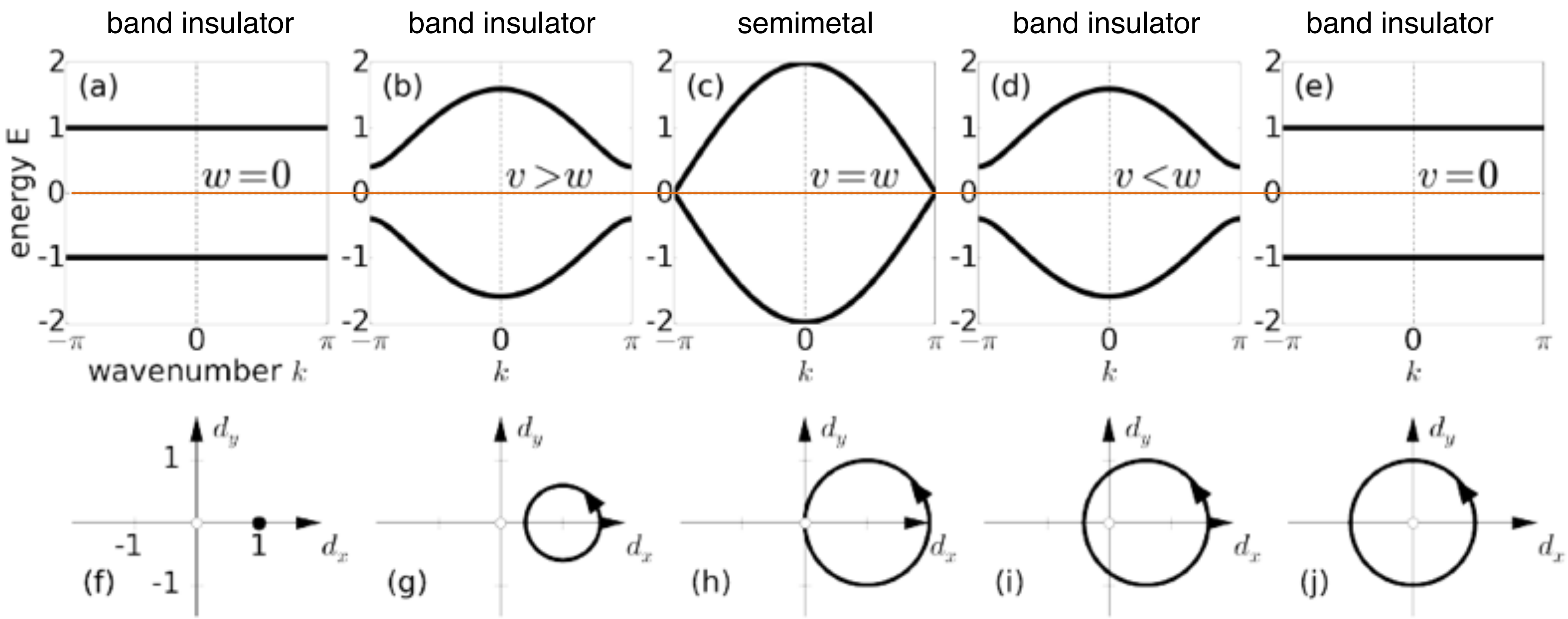


(a)
(b)
(c)
(d)
(e)
energy E
wavenumber k
k
$w = 0$
$v > w$
$v = w$
$v < w$
$v = 0$
2
1
0
-1
-2
$-\pi$
π

"half-filling": $E_{\text{Fermi}} = 0$

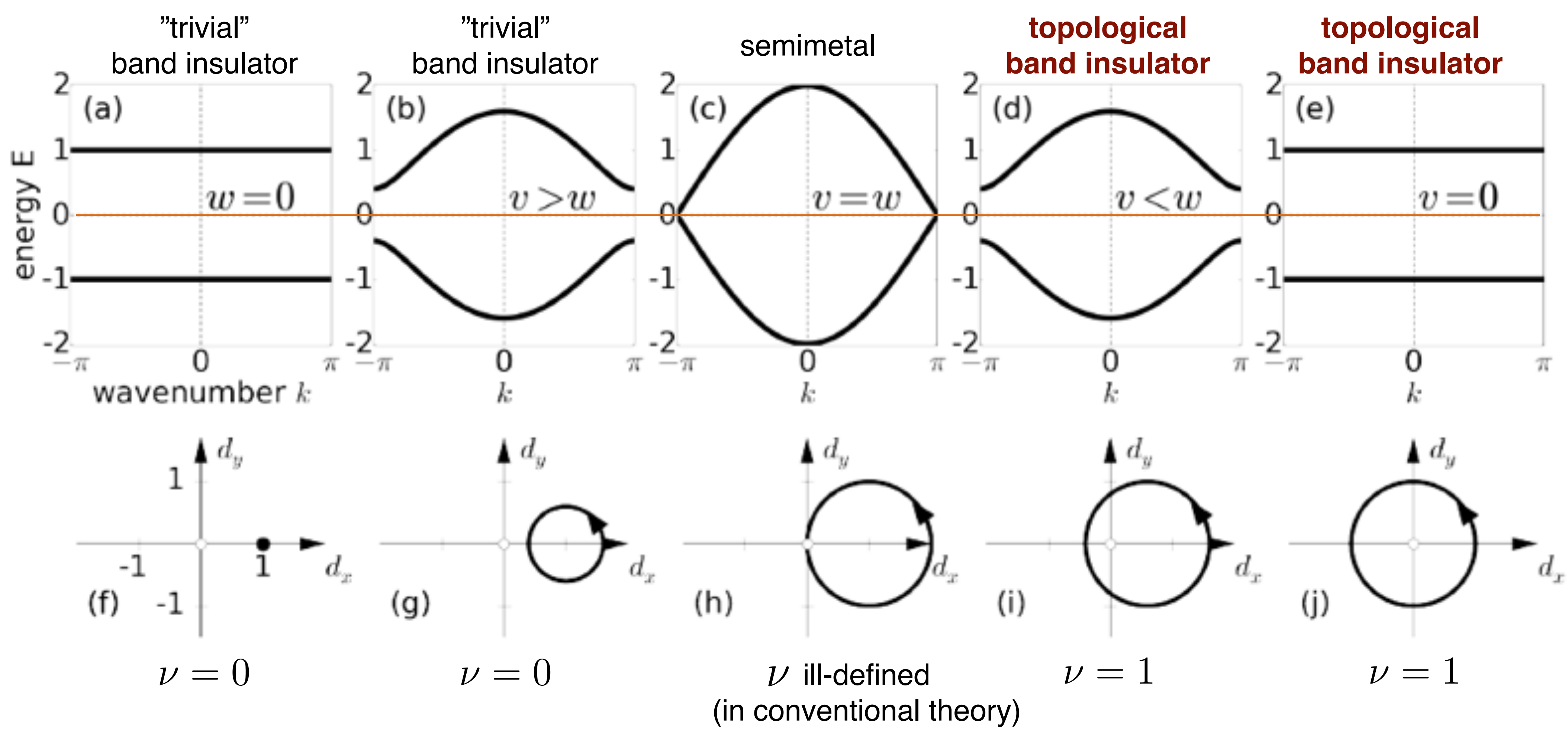


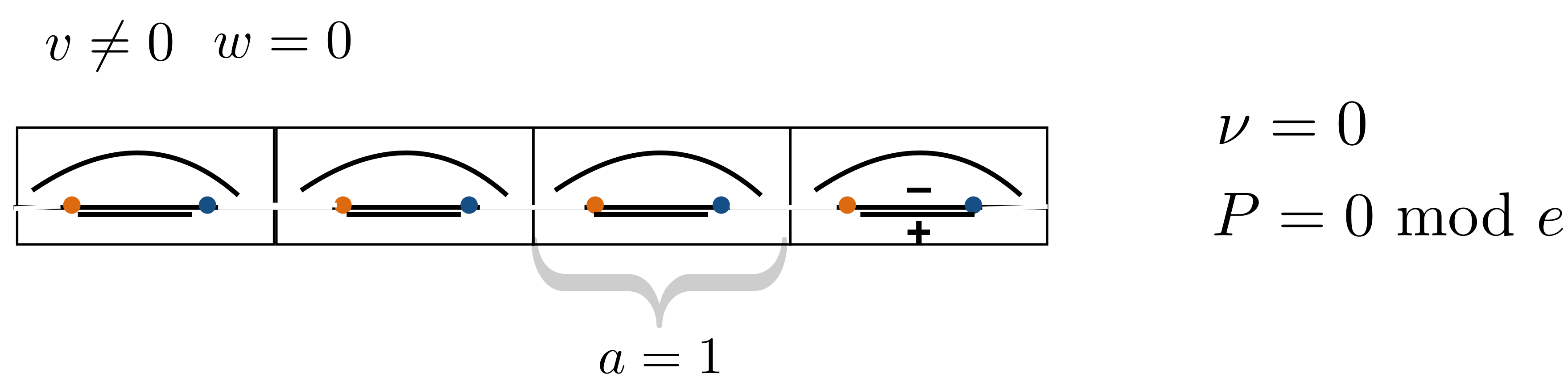
"half-filling": $E_{\text{Fermi}} = 0$



$\nu = 0$ $\nu = 0$ ν ill-defined (in conventional theory) $\nu = 1$ $\nu = 1$

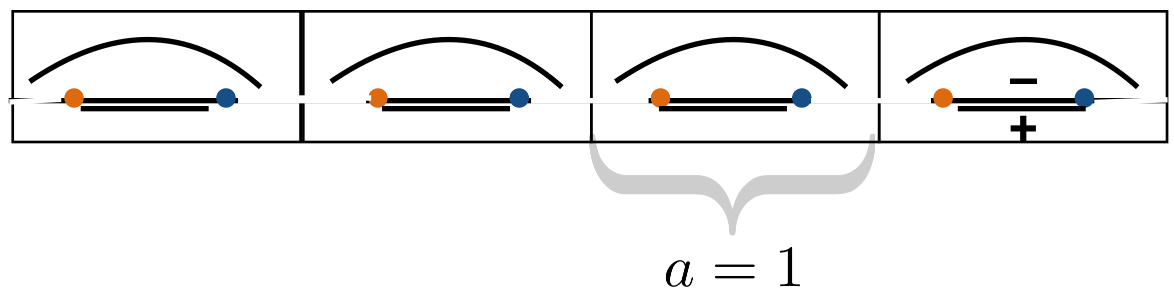
"half-filling": $E_{\text{Fermi}} = 0$





$v \neq 0 \quad w = 0$
$a = 1$
$\nu = 0$
$P = 0 \bmod e$

$v \neq 0 \quad w = 0$

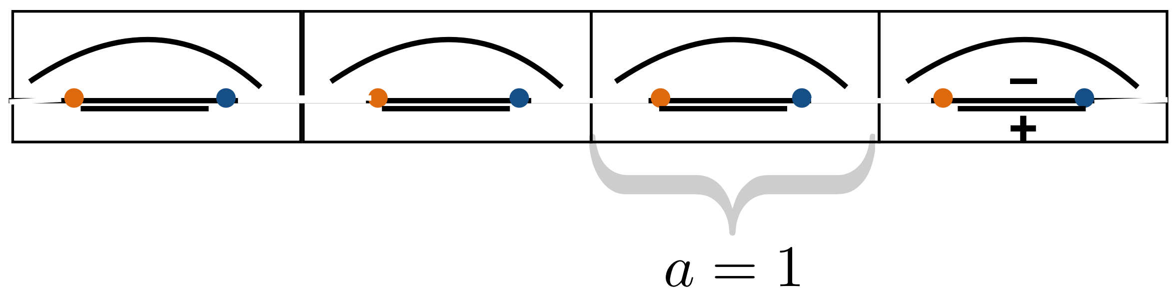


$\nu = 0$

$P = 0 \ \cancel{\mathrm{mod}\ e}$

"gauge fixing"

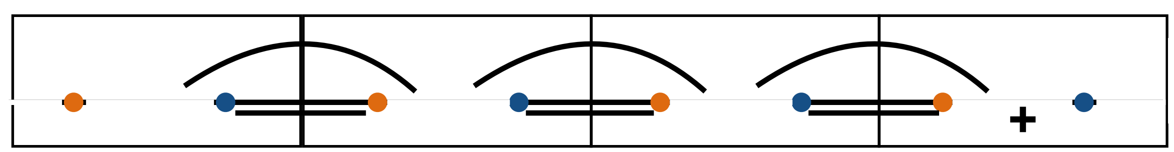
$v \neq 0 \quad w = 0$



$\nu = 0$

$\cancel{P = 0 \bmod e}$

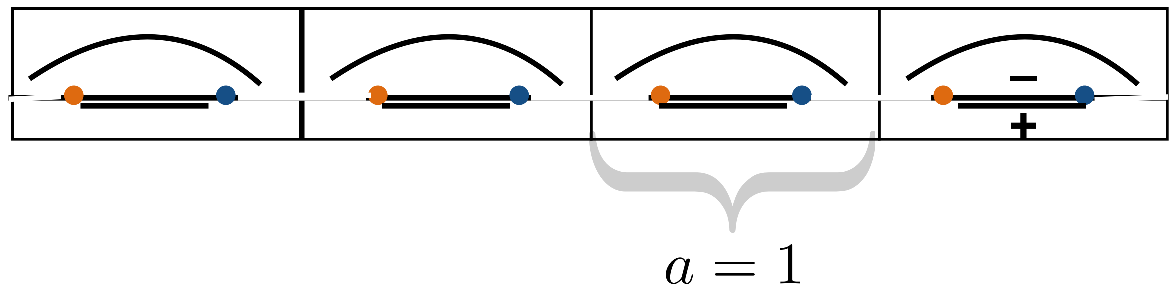
$w \neq 0 \quad v = 0$



$\nu = 1$

$\cancel{P = e/2 \bmod e}$

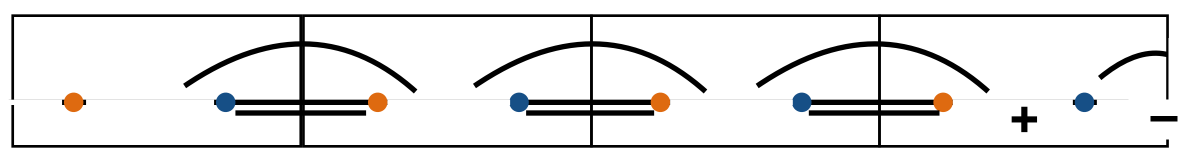
$v \neq 0 \quad w = 0$



$\nu = 0$

$P = 0 \cancel{\bmod e}$

$w \neq 0 \quad v = 0$



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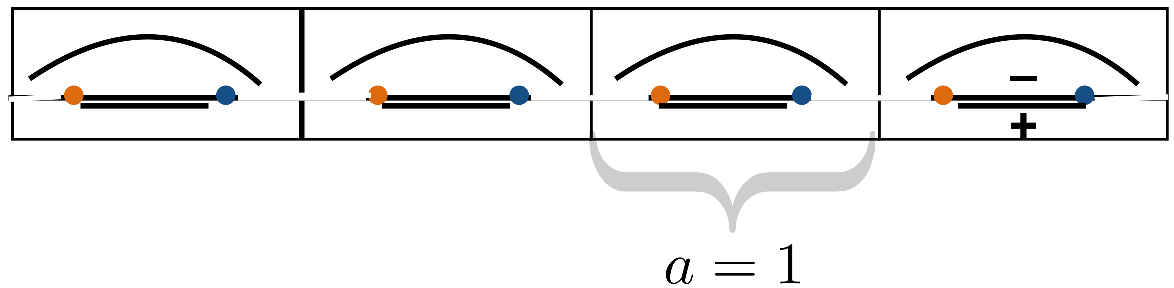
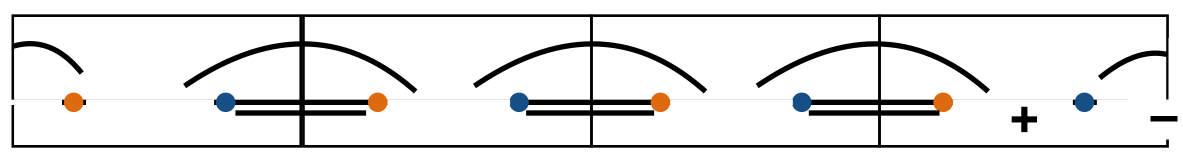


$\nu = 0$

$P = 0 \ \cancel{\mathrm{mod}\ e}$

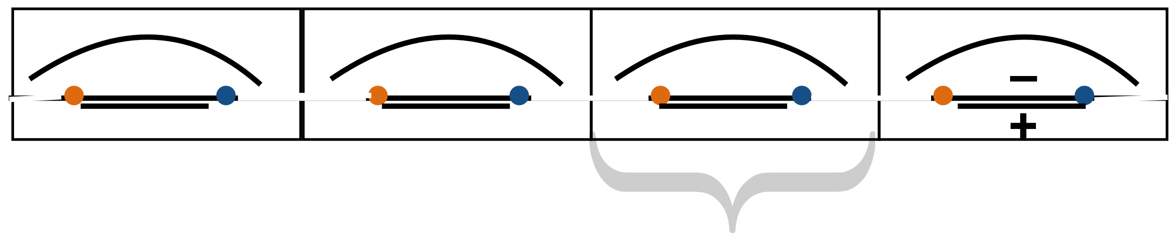
$w \neq 0 \quad v = 0$



$\nu = 1$

$P = e/2 \ \cancel{\mathrm{mod}\ e}$

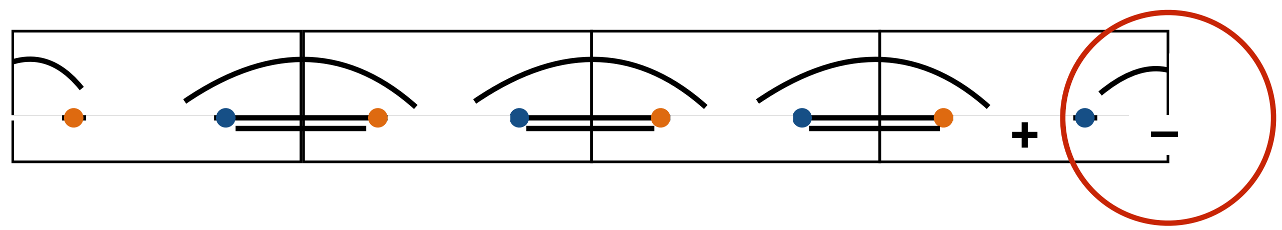
$v \neq 0 \quad w = 0$



$\nu = 0$

$P = 0 \cancel{\bmod e}$

$w \neq 0 \quad v = 0$



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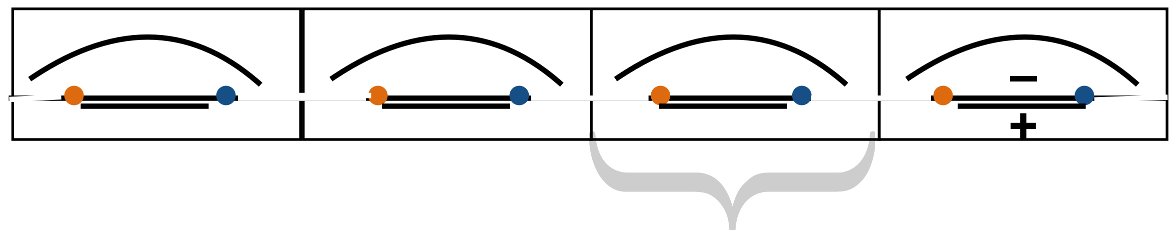
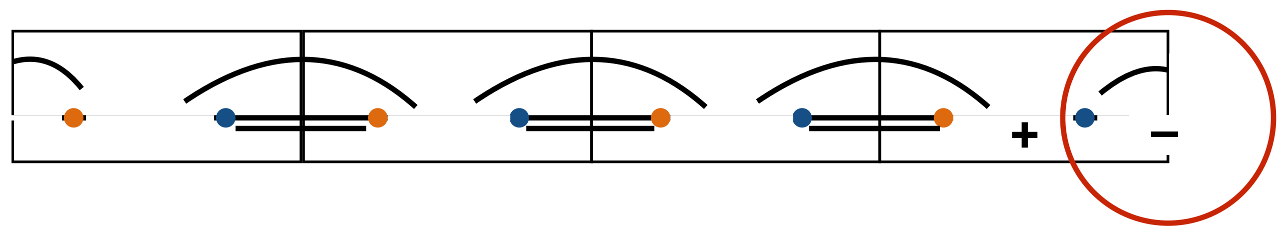


$\nu = 0$

$P = 0 \ \cancel{\bmod e}$

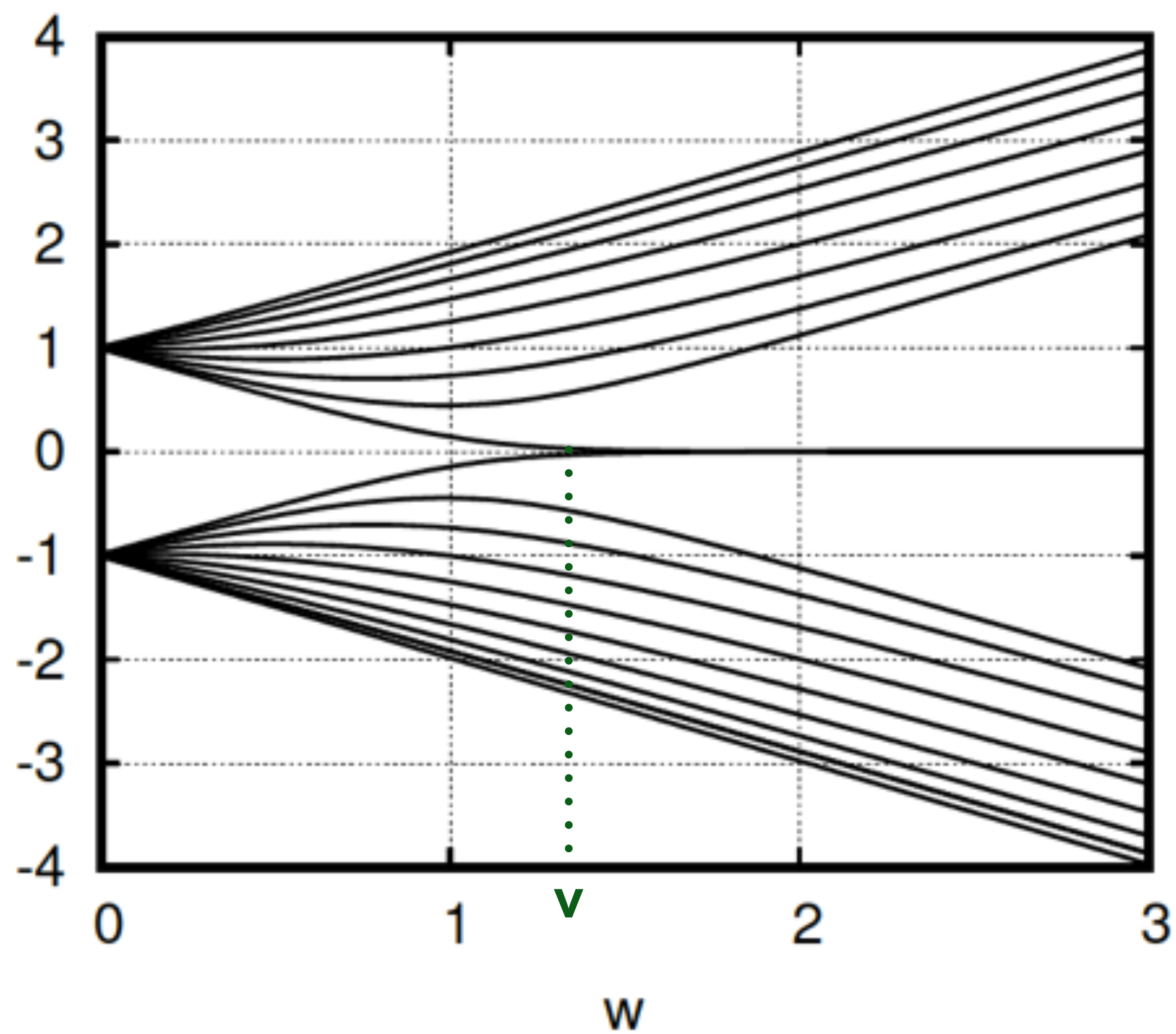
$w \neq 0 \quad v = 0$



$\nu = 1$

$P = e/2 \ \cancel{\bmod e}$

This boundary state is robust against perturbations which respect certain symmetries.
This is THE hallmark of ”symmetry-protected topological phases”!



4
3
2
1
0
-1
-2
-3
-4
0
1
v
2
3
w

"Periodic table" of symmetry-protected topological systems (noninteracting fermions)

class	T	C	S	0	1	2	3
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0
BDI	+	+	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
D	0	+	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	−	+	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	−	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	−	−	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	−	0	0	0	$2\mathbb{Z}$	0
CI	+	−	1	0	0	0	$2\mathbb{Z}$

Anderson Localization (Schnyder, Ryu, Furusaki, Ludwig, 2008; 2009; 2010)
Topology (K-Theory) (Kitaev, 2009)

review: C.-K. Chiu *et al.,* Rev. Mod. Phys. **88**, 035005 (2016)

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BDI	+	+	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
D	0	+	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	−	+	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	−	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	−	−	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	−	0	0	0	$2\mathbb{Z}$	0
CI	+	−	1	0	0	0	$2\mathbb{Z}$

example:
integer quantum Hall effect
topological invariant: *Chern number*

example:
SSH model
topological invariant: *winding number*

Anderson Localization (Schnyder, Ryu, Furusaki, Ludwig, 2008; 2009; 2010)
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D	0	+	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	−	+	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	−	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	−	−	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	−	0	0	0	$2\mathbb{Z}$	0
CI	+	−	1	0	0	0	$2\mathbb{Z}$

example:
SSH model
topological invariant: *winding number (when allowing only perturbations that have* ***all*** *the symmetries of the unperturbed Hamiltonian!)*

Anderson Localization (Schnyder, Ryu, Furusaki, Ludwig, 2008; 2009; 2010)
Topology (K-Theory) (Kitaev, 2009)

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AI	$+$	0	0	$\mathbb{Z}$	0	0	0
BDI	$+$	$+$	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
D	0	$+$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	$-$	$+$	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	$-$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	$-$	$-$	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	$-$	0	0	0	$2\mathbb{Z}$	0
CI	$+$	$-$	1	0	0	0	$2\mathbb{Z}$

another example:
SSH model with a perturbation that
only respects time-reversal symmetry
no *topological phase!*

Anderson Localization (Schnyder, Ryu, Furusaki, Ludwig, 2008; 2009; 2010)
Topology (K-Theory) (Kitaev, 2009)

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D	0	$+$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	$-$	$+$	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	$-$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	$-$	$-$	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	$-$	0	0	0	$2\mathbb{Z}$	0
CI	$+$	$-$	1	0	0	0	$2\mathbb{Z}$

By **adding** *space group symmetries* to T, C, and S (eg. mirror symmetry), a topologically trivial phase (like the D=1 AII phase) may split into a trivial and a topologically nontrivial phase. *Crystalline topological insulators!*

Anderson Localization (Schnyder, Ryu, Furusaki, Ludwig, 2008; 2009; 2010)
Topology (K-Theory) (Kitaev, 2009)

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